6.5 - Properties of Logarithms

Warmup

Expand the expression

1. $\log_3(xy)$

$\log_3 x + \log_3 y$

2. $\log_4(rst)$

$\log_4 r + \log_4 s + \log_4 t$

3. $\log_2\left(\frac{x}{r}\right)$

$\log_2 x - \log_2 r$

4. $\log_b\left(\frac{\sqrt{x}}{p}\right)$

$\frac{1}{2}\log_b x - \log_b p$

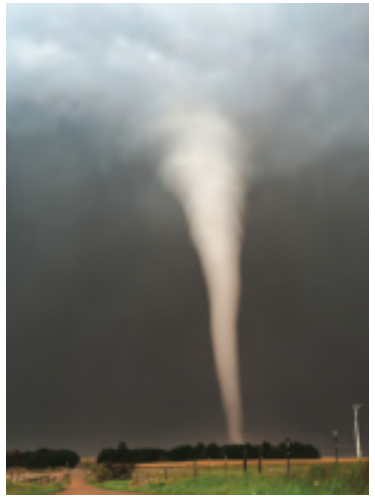
5. $\log_3(5\sqrt[3]{a})$

$\log_3 5 + \frac{1}{3}\log_3 a$

6. $\log_{10}(ac)^2$

$2\log_{10} a + 2\log_{10} c$

Chapter 6 Exponential and Logarithmic Functions



1. Exponential Growth and Decay Functions
2. The Natural Base e
3. Logarithms and Logarithmic Functions
4. Transformations of Exponential and Logarithmic Functions
5. Properties of Logarithms
6. **Solving Exponential and Logarithmic Equations**
7. Modeling with Exponential and Logarithmic Functions

6.6 - Solving Exponential and Logarithmic Equations

Solve

$$3^{\log_3 x} = 3x - 4$$

$$x = 3x - 4$$

$$x = 2$$

Simplify

1. $6^{\log_6(x+2)} = 2x - 6$

$x = 8$

2. $4^{\log_2(x+1)} = 25$

$x = 4, \cancel{-6}$

6.6 - Solving Exponential and Logarithmic Equations

Solve the expression

$$2e^{12x} = 17 \qquad e^{12x} = \frac{17}{2}$$

BTW

$$\log_e = \ln$$

$$12x = \ln 8.5 \qquad x = 0.17833$$

Practice

1) $$\frac{50}{1 + e^{-x}} = 4$$

$$x = -\ln \frac{23}{2} \approx -2.4423$$

2) $$e^{2x} - 3e^{x} + 2 = 0$$

$$x = \{0,\ \ln 2\} \approx \{0,\ 0.69315\}$$

6.6 - Solving Exponential and Logarithmic Equations

Solve the expression

$\log x + \log(x-1) = \log(4x)$

$\log x(x-1) = \log(4x)$

$\log(x^2 - x) = \log(4x)$

$x^2 - x = 4x$

$x^2 - 5x = 0$

$x(x-5) = 0$

$x = \cancel{0}, 5$

Practice

1) $\log_2 3 + \log_2 x = \log_2 5 + \log_2(x-2)$ $x = 5$

2) $\log_2(x^2 - x - 2) = 2$ $x = -2, 3$

6.6 - Solving Exponential and Logarithmic Equations

Solving Equations

$$12^{2x} = 48$$

$$\log_{12} 12^{2x} = \log_{12} 48$$

$$2x = \log_{12} 48$$

$$x = \frac{\log 48}{2 \log 12} \approx 0.7789$$

$$\log_e x = \ln x$$

$$\ln e^5 = 5$$

$$\ln 4 - \ln 10 = \ln \frac{4}{10}$$

$$= \ln 0.4 \approx -0.91629$$

Practice - to three decimal places

1. $\ln 3x = 2$

$x \approx 2.463$

2. $\ln 3 + \ln 2x = \ln 36$

$x = 6$

3. $3e^{2x} + 2 = 50$

$x \approx 1.386$

6.6 - Solving Exponential and Logarithmic Equations

Exponential Equations

$$3^{2x} - 2 \cdot 3^x - 8 = 0$$

$$z = 3^x$$

$$z^2 - 2z - 8 = 0$$

$$(z - 4)(z + 2) = 0$$

$$z = 4, \cancel{-2}$$

$$4 = 3^x$$

$$\log_3 4 = \log_3 3^x$$

$$\log_3 4 = x$$

$$\frac{\log 4}{\log 3} = x$$

$$x \approx 1.2619$$

Practice - leave in terms of log or ln

1. $3^{2x} - 5 \cdot 3^x + 4 = 0$

$0, \log_3 4$

2. $e^{2x} - e^x - 6 = 0$

$\ln 3$

3. $5^{2x+1} + 3 \cdot 5^x - 2 = 0$

$\log_5 \frac{2}{5}$

6.6 - Solving Exponential and Logarithmic Equations

Exponential Equations

$$3^{2x} - 2 \cdot 3^x - 8 = 0$$

$$z = 3^x$$

$$z^2 - 2z - 8 = 0$$

$$(z - 4)(z + 2) = 0$$

$$z = 4, \cancel{-2}$$

$$4 = 3^x$$

$$\log_3 4 = \log_3 3^x$$

$$\log_3 4 = x$$

$$\frac{\log 4}{\log 3} = x$$

$$x \approx 1.2619$$

Practice - leave in terms of log or ln

1. $3^{2x+1} - 4 \cdot 3^x - 4 = 0$

$\log_3 2$

2. $5^{2x-1} - 2 \cdot 5^x + 5 = 0$

1

3. $5^{2x+2} - 3 \cdot 5^{x+1} + 2 = 0$

$\log_5 \frac{2}{5}, -1$

6.6 - Solving Exponential and Logarithmic Equations

Exponential Equations

$$e^x + e^{-x} = 5$$

$$e^x(e^x + e^{-x} = 5)$$

$$e^{2x} + 1 = 5e^x$$

$$e^{2x} - 5e^x + 1 = 0$$

$$e^x = \frac{5 \pm \sqrt{21}}{2} \approx 0.2087121525 \approx 4.7912878475$$

$$x \approx -1.5667992371$$

$$x \approx 1.566799237$$

Practice

1. $e^x - 6e^{-x} = 1$

$\ln 3 \approx 1.0986$

2) $\dfrac{e^x + e^{-x}}{e^x - e^{-x}} = 3$

$\frac{1}{2}\ln 2 \approx 0.34657$

6.6 - Solving Exponential and Logarithmic Equations

Exponential Equations

$$e^x + e^{-x} = 5$$

$$e^x(e^x + e^{-x} = 5)$$

$$e^{2x} + 1 = 5e^x$$

$$e^{2x} - 5e^x + 1 = 0$$

$$e^x = \frac{5 \pm \sqrt{21}}{2} \approx 0.2087121525 \approx 4.7912878475$$

$$x \approx -1.5667992371$$

$$x \approx 1.566799237$$

Practice

1. $3^{-2x} - 4 \cdot 3^{-x} + 3 = 0$

$x = 0, -1$

2) $5^{6x} - 2 \cdot 5^{3x} - 3 = 0$

$\frac{1}{3}\log_5 3 \approx 0.2275$